STANDARD 10 — ESTIMATION

K-12 Overview

All students will use a variety of estimation strategies and recognize situations in which estimation is appropriate.

Descriptive Statement

Estimation is a process that is used constantly by mathematically capable adults, and that can be mastered easily by children. It involves an educated guess about a quantity or a measure, or an intelligent prediction of the outcome of a computation. The growing use of calculators makes it more important than ever that students know when a computed answer is reasonable; the best way to make that decision is through estimation. Equally important is an awareness of the many situations in which an approximate answer is as good as, or even preferable to, an exact answer.

Meaning and Importance

As used in this standard, estimation is the process of determining approximate values in a variety of situations. Estimation strategies are used universally throughout daily life, but an examination of the mathematics curriculum of the past leads to the view that the strength of mathematics lies in its exactness, in the ability to determine the “right” answer. The growing use of calculators in the classroom requires greater emphasis on determining whether the answer given by a calculator or paper-and-pencil method is reasonable, a process that requires estimation ability, but efforts in support of this goal have been minimal compared to the time devoted to getting that one right answer. As a result, students have developed the notion that exactness is always preferred to estimation and their potential development of intuition may have been hindered with unnecessary calculations and detail.

People who use mathematics in their lives and careers find estimation to be preferable to the use of exact numbers in many circumstances. Frequently, it is either impossible to obtain exact answers or too expensive to do so. *An air conditioning salesperson preparing a bid would be wasting time and money by measuring rooms exactly. Astronomers attempting to determine movements of celestial objects cannot obtain precise measurements.* Many people use approximations because it is easier than using exact numbers. *Shoppers, for example, use approximations to determine whether they have sufficient funds to purchase items. Travelers use rough estimates of time, distance, and cost when planning trips*. Commonly reported data often use levels of precision which have been accepted as appropriate, even though they may not be considered “exact.” *Astronomers usually report information to two significant digits, and batting averages for baseball players are always reported as three-place decimals*.

K-12 Development and Emphases

Part of being functionally numerate requires expertise in using **estimation with computation**. Such facility demands a strong sense of number as well as a mastery of the basic facts, an understanding of the properties of the operations as well as their appropriate uses, and the ability to compute mentally. As these skills and understandings are developed throughout the mathematics curriculum, students should have frequent opportunities to develop methods for obtaining estimates, and to recognize that estimation is useful. Estimation can help determine the correct answer from a set of possible answers, and establish the **reasonableness of answers**. Ideally, students should have an idea of the approximate size of an answer; then, if they recognize that the result they have obtained is incorrect, they can immediately rework the problem. This ability becomes increasingly important as students use calculators more and more.

Instruction in estimation has traditionally focused on the use of rounding. There are times when rounding is an appropriate process for finding an estimate, but this standard emphasizes that it is only one of a variety of processes. Computational **estimation strategies** are a new and important component in the curriculum. *Clustering, front-end digits, compatible numbers*, and other strategies are all helpful to the skillful user of mathematics, and can all be mastered by young students. Selection of the appropriate strategy to use depends on the setting, and the numbers and operations involved. Students should realize that there is not necessarily a "right" answer when estimating; different techniques may yield different estimates, and that is quite acceptable.

The foregoing discussion describes a new emphasis on the use of estimation in computational settings, but students should also be thoroughly comfortable with the use of **estimation in measurement**. Students should develop the ability to estimate measures such as length, area, volume, and angle size visually as well as through the use of personal referents, such as the width of a finger or the length of a pace. Measurement is rich with opportunities to develop an understanding that estimates are often used to determine approximate values which are then used in computations, and that results so obtained are not exact but fall within **a range of tolerance**.

Estimation should be emphasized in many other areas of the mathematics curriculum in addition to the obvious uses in numerical operations and measurement. Within statistics, for example, it is often useful to estimate measures of central tendency for a set of data; estimating probabilities can help a student determine when a particular course of action would be advisable; problem situations related to algebraic concepts provide opportunities to estimate rates such as slopes of lines and average speed; and working with sequences in algebra and increasing the number of sides of a regular polygon in geometry yield opportunities to estimate limits.

IN SUMMARY, estimation is a combination of content and process. Students' abilities to use estimation appropriately in their daily lives develops as they have regular opportunities to explore and construct estimation strategies and as they acquire an appreciation of its usefulness in the solution of problems.

***NOTE**: Although each content standard is discussed in a separate chapter, it is not the intention that each be treated separately in the classroom. Indeed, as noted in the Introduction to this Framework, an effective curriculum is one that successfully integrates these areas to present students with rich and meaningful cross-strand experiences.*

Standard 10 — Estimation — Grades K-2

Overview

As indicated in the K-12 Overview, students' ability to use estimation appropriately in their daily lives develops as they focus on the **reasonableness of answers**, explore and construct **estimation strategies**, and **estimate measurements**, **quantities**, **and the results of computation**.

One of the estimation emphases for very young children is the development of the idea that guessing is an important and exciting part of mathematics. The teacher must employ sound management practices which ensure that everyone's guess is important and which encourage student risk-taking and sharing of ideas about how their guesses were determined. When first asked to guess an answer, many students will give nonsense responses until they establish appropriate experiences, build their sense of numbers, and develop informal strategies for creating a guess. Children begin to make reasonable estimates when the situations involved are relevant to their immediate world. Building on comparisons of common objects and using personal items to build a sense of lengths, weights, or quantities helps children gain confidence in their guessing. As children communicate with each other about how guesses are formulated they begin to develop informal **strategies for estimation**.

Estimation with computation is as important at these early grade levels as it is at all the other grade levels. Estimation of sums and differences should be a part of the computational process from the very first activity with any sort of computation. Children should regularly be asked *About how many do you think there will be in all?* or *About what do you think the difference is?* or *About how many do you think will be left?* in the standard addition and subtraction settings. These questions are appropriate whether or not exact computations will be done. Children should understand that, sometimes, the estimate will be an accurate enough number to serve as an answer. At other times, an exact computation will need to be done, either mentally, with paper-and-pencil, or with a calculator to arrive at a more precise answer. The particular procedure to be used is dependent on the setting and the problem.

One of the most useful computational **estimation strategies** at these grade levels also reinforces an important place value idea. Students should understand that in two-digit numbers the tens digit is much more meaningful than the ones digit in contributing to the overall value of the number. A reasonable approximation, then, of a two-digit sum or difference can always be made by considering only the tens digits and ignoring the ones. This strategy is referred to as *front end estimation* and is used with larger numbers as well, although then the first two digits may be used. It is the main estimation strategy that many adults use.

Standard 10 — Estimation — Grades K-2

Indicators and Activities

The cumulative progress indicators for grade 4 appear below in boldface type. Each indicator is followed by activities which illustrate how it can be addressed in the classroom in kindergarten and in grades 1 and 2.

Experiences will be such that all students in grades K-2:

1. **Judge without counting whether a set of objects has less than, more than, or the same number of objects as a reference set.**

   - Students place various amounts of counters or other small objects in individual plastic bags. Working in groups of four students, the children choose one bag to be the reference set and judge whether each of the other bags has more than, less than, or the same as the reference set. Initially, they should try to make the judgments without counting. The teacher observes the groups as they work, making notes about the students' progress.
   - Young children benefit from frequently comparing sets of objects to some given number. For example, given sets of colored chips arranged on a table, they should name which sets have more than five and which have less than five.
   - Students play the card game *War* with a set of cards without numerals (i.e., cards which only show sets of hearts, clubs, diamonds, or spades). Students will easily distinguish between the 7's and the 3's, but will be reluctant to make judgments about closer numbers like the 4's and 5's without counting. As they play more often, however, their ability to distinguish will visibly improve. They may also begin to notice patterns involving even and odd numbers on their own.
   - Students learn to recognize certain arrangements of dots or stars as representing certain numbers. Using flashcards, they estimate the number of dots or stars, and then count to check their estimates.
   - After reading *Ten Black Dots* by Donald Crews, students make up their own uses for 1-10 black dots. They use adhesive dots to create their own books that include uses for each of the numbers from 1 through 10. They then estimate and count the total number of dots they actually use for their book. (The total might surprise them: 55.)
   - As an assessment of students' ability to judge without counting, the teacher puts some counters (more than five) on the overhead projector, turns it on for a few seconds, and then asks the students to write whether the number of counters shown is closer to 10 or to 20.

2. **Use personal referents, such as the width of a finger as one centimeter, for estimations with measurement.**

   - Students estimate lengths of pieces of spaghetti, yarn, paper, pencils, paper clips, etc., using suggested non-standard personal units such as width of thumb, length of a foot, and so on. They note that different students get different "right" answers.
   - As standard units like foot and centimeter are introduced, students are challenged to find

some part of their body or some personal action that is about that size at this point in their growth. For instance, they may decide that the width of their little finger is almost exactly one centimeter or the length of one baby step is one foot.

- Students use their self-discovered personal body referents to estimate the measures of various classroom objects like the length of the blackboard or the width of a piece of paper. They compare their answers, noting that when larger units are used, the estimated answer is a smaller number; and when smaller units are used, the estimated answer is a larger number.

3. Visually estimate length, area, volume, or angle measure.

- Students look at a quantity of sand, salt, flour, water, macaroni, corn, or popcorn and estimate how many times it could fill up a specified container.
- Students estimate how many pieces of notebook paper it would take to cover a given area such as the blackboard, or a portion of the classroom floor.
- Students regularly estimate lengths using a variety of non-standard units such as *my feet, Unifix cubes, paper clips,* and *orange Cuisenaire Rods.* They then measure to verify or revise their estimates.
- Students begin to develop an understanding of angle measure by making right-hand (or left-hand) turns repeatedly to turn completely around. They also compare angles to right-angle "corners," and decide whether an angle is more than or less than a "corner."
- Students note that there are 12 numbers on a clock face and discuss how far each hand moves in an hour. They note that each hand moves in a circle, but that the hour hand moves much more slowly.
- Students work through the *Will a Dinosaur Fit?* lesson that is described in the First Four Standards of this *Framework.* They determine the size of the room, hear their classmates' presentations about the dinosaurs, and then as a whole class activity estimate which dinosaurs, and how many of them, might fit into the room.

4. Explore, construct, and use a variety of estimation strategies.

- Students are asked if a sixty-seat bus will be adequate to take the two first grade classes on their field trip. After it is known that there are 23 children in one class and 27 in the other, individuals volunteer their answers and give a rationale to support their thinking; *front end estimation* should lead to the conclusion that the total number of students is between 40 and 60. A discussion might be directed to the question of whether an exact answer to the computation was needed for the problem.
- Students are shown a glass jar filled with about eighty marbles and asked to estimate the number in the jar. In small groups, they discuss various approaches to the problem and strategies they can use. Each group shares one strategy with the class, and the estimate that resulted. The teacher makes notes about students' work throughout the activity.
- Second grade students can be challenged to estimate the total number of students in the school. They will need to talk informally about the average number of students in each class, the number of classes in a grade level, and the number of grade levels in the school. They might then use calculators to get an answer, but the result, even though the exact answer to a

computation, is still an estimate to the original problem. They discuss why that is so.

- Primary-grade students explore the meanings of comparison words by listening to *How Many is Many?* by Margaret Tuten. They compare *big* and *small, long* and *short,* a *lot* and a *few*. They list how many pieces of candy would be a few and how many pieces would be many, eventually reaching general agreement, perhaps on 5 as a few. Then they consider whether 5 teaspoons of medicine would be a few.

5. Recognize when estimation is appropriate, and understand the usefulness of an estimate as distinct from an exact answer.

- Given a pair of real-life situations, students determine which situation in the pair is the one for which estimation is a good approach and which is the one that probably requires an exact answer. One such pair, for example, might be: *sharing a bag of peanuts among 3 friends* and *paying for 3 tickets at the movie theater*.

- Given a set of cartoons with home-made mathematical captions, first graders decide which of the cartoon characters arrived at exact answers and which got estimates. Two of the cartoons might show an adult and a child looking at a jar of jellybeans and the captions might read: *Susie guessed that there were 18 jellybeans left in the jar* and *Susie's mom counted the 14 jellybeans left in the jar.*

- Students read or listen to newspaper headlines and discuss which involve exact numbers and which might be estimates.

6. Determine the reasonableness of an answer by estimating the result of operations.

- Students are regularly asked if their answer makes sense in the context of the problem they were solving. They respond with full sentences explaining what they were asked to find and why the numerical answer they found fits the context reasonably, that is, why it *could be* the answer. For example, first graders might be asked to decide if their answer to the following problem makes sense: *Mary made 27 cookies and Jose made 15. How many cookies did they make in all?* Some responses might indicate that the answer should be more than $20 + 10 = 30$ and less than $30 + 20 = 50$. Other students might say that they know that *25* and *15* is *40*, so the answer should be a little more.

- Students estimate *reasonable* numbers of times that particular physical feats can be performed in one minute. For example: *How many times can you bounce a basketball in a minute? How many times can you hop on one foot in a minute? How many times can you say the alphabet in a minute?* and so on. Other students judge whether the estimates are reasonable or unreasonable and then the tasks are performed and the actual counts made.

- Second-grade students are given a set of thirty cards with two-digit addition problems on them. In one minute, they must sort the cards into two piles: those problems whose answers are greater than 100 and those less than 100. The correct answers can be on the backs of the cards to allow self-checking after the task is completed.

- Second-grade students are given a page of addition or subtraction problems in a multiple choice format with 4 possible answers for each problem. Within some time period which is much too short for them to do the computations, students are asked to choose the most reasonable answer from each set of four.

7. **Apply estimation in working with quantities, measurement, time, computation, and problem solving.**

    - Students have small pieces of yarn of slightly different lengths ranging from 2 to 6 inches. Each student first estimates the number of his or her pieces it would take to match a much longer piece — about 30 inches long — and then actually counts how many. Then they use their individual pieces to measure other objects in the room. Each child is responsible for estimating the lengths in terms of his or her own yarn, but they can use evidence from other children's measuring to help make their own estimates.
    - Students regularly estimate in situations involving classroom routines. For example, at snack time, they may guess how many cups can be filled by each can of juice or how many crackers each student will get if all of the crackers in the box are given out.
    - Kindergartners always have fun deciding which color is best represented in a group of multi-colored objects. Good examples of such an activity would be choosing the color that shows up most often (or least often) in bags of M&M's, in handfuls of small squares of colored paper, or in a jar full of marbles. After everyone has committed to a guess, the children can sort the objects and count each color. They can then make bar graphs to show the distribution of the different colors.
    - Students use Tana Hoban's photographs in *Is It Larger? Is It Smaller?* as a starting point for investigating and comparing quantities and measures in their classroom. For example, on one page, three vases are shown filled with three different kinds of flowers. The reader must decide which objects to compare, such as the vases, before ordering them — from tallest to shortest and/or by volume.

References

Crews, Donald. *Ten Black Dots.* New York: Greenwillow, 1986.

Hoban, Tana. *Is It Larger? Is It Smaller?* New York: Greenwillow, 1985.

Tuten, Margaret. *How Many is Many?* Chicago: Children's Press, 1970.

On-Line Resources

http://dimacs.rutgers.edu/nj_math_coalition/framework.html/

The *Framework* will be available at this site during Spring 1997. In time, we hope to post additional resources relating to this standard, such as grade-specific activities submitted by New Jersey teachers, and to provide a forum to discuss the *Mathematics Standards*.

Standard 10 — Estimation — Grades 3-4

Overview

As indicated in the K-12 Overview, students' ability to use estimation appropriately in their daily lives develops as they focus on the **reasonableness of answers**, explore and construct **estimation strategies,** and **estimate measurements, quantities, and the results of computation.**

For this type of development to occur, the atmosphere established in the classroom ought to assure everyone that their estimates are important and valued. Children should feel comfortable taking risks, and should understand that an explanation and justification of estimation strategies is a regular part of the process. Third- and fourth-graders, for the most part, should be beyond just "guessing." As children communicate with each other about how their estimates are formulated, they further develop their personal bank of **strategies for estimation.**

Students should already feel comfortable with estimation of sums and differences from their work in earlier grades. Nonetheless, they should regularly be asked *About how many do you think there will be in all?* or *About what do you think the difference is?* or *About how many do you think will be left?* in the standard addition and subtraction settings. These questions are appropriate whether or not exact computations will be done. As the concepts and the related facts of multiplication and division are introduced through experiences that are relevant to the child's world, **estimation with computation** must again be integrated into the development and practice activities.

One of the most useful computational **estimation strategies** in these grade levels also reinforces an important place-value idea. Students should understand that in multi-digit whole numbers the larger the place value, the more meaningful the digit in that position is in contributing to the overall value of the number. A reasonable approximation, then, of a multi-digit sum or difference can always be made by considering only the leftmost places and ignoring the others. This strategy is referred to as *front end estimation* and is the main estimation strategy that many adults use. In third and fourth grades, it should accompany the traditional *rounding* strategies.

Children should understand that, sometimes, the estimate will be accurate enough to serve as an answer. At other times, an exact computation will need to be done, either mentally, with paper-and-pencil, or with a calculator to arrive at a more precise answer. The particular procedure to be used is dependent on the setting and the problem. Also at this level, estimation must be an integral part of the development of concrete, algorithmic, or calculator approaches to multi-digit computation. Students must be given experiences which clearly indicate the importance of formulating an estimate *before* the exact answer is calculated.

In third and fourth grades, students are developing the concepts of a thousand and then of a million. Many opportunities arise where estimation of quantity is easily integrated into the curriculum. Many *what if* questions can be posed so that students continue to use estimation skills to determine practical answers.

Standard 10 — Estimation — Grades 3-4

Indicators and Activities

The cumulative progress indicators for grade 4 appear below in boldface type. Each indicator is followed by activities which illustrate how it can be addressed in the classroom in grades 3 and 4.

Building upon knowledge and skills gained in the preceding grades, experiences in grades 3-4 will be such that all students:

1. Judge without counting whether a set of objects has less than, more than, or the same number of objects as a reference set.

- Students estimate the numbers represented by groups of base ten blocks or bundles of popsicle sticks. For example, one set might consist of 1 hundred, 6 tens, and 3 ones, and the other 0 hundreds, 17 tens, and 7 ones. Students first estimate which is more without arranging the blocks or counting them and then they determine the correct answer. These kinds of proportional models allow the "quantity of wood" to be proportional to the actual size of the number.

- Students read *The Popcorn Book* by Tomie dePaola and make estimates with popcorn. For example, they might consider two quantities of popcorn, one popped and one unpopped. *Which contains the largest* ***number*** *of kernels?* They might also predict how many measuring cups the unpopped popcorn will fill once it is popped and find out the result after popping the popcorn. (The total number of cups made may be surprising.)

2. Use personal referents, such as the width of a finger as one centimeter, for estimations with measurement.

- Students estimate the height of a classmate in inches or centimeters by standing next to him or her and using their own known height for comparison.

- As standard units like yard and half-inch are introduced, students are challenged to find some part of their body or some personal action that is about that size at this point in their growth. For instance, they may decide that the width of their little finger is almost exactly one half-inch or the length of two *giant steps* is one yard.

- Students measure the width of their handspan in centimeters (from thumb tip to little finger tip with the hand spread as far as possible) and then use the knowledge of its width to estimate the metric measures of various classroom objects by counting the number of handspans across and multiplying by the number of centimeters.

3. Visually estimate length, area, volume, or angle measure.

- Students estimate the number of 3" x 5" cards it would take to cover their desktops, a floor tile, and the blackboard. They describe the process they used in writing, which is then read by the teacher to determine the students' progress.

- Students work through the *Tiling a Floor* lesson that is described in the First Four Standards of this *Framework*. They estimate how many of their tiles it would take to cover one sheet of paper, and compare their answers to the actual number needed.

- Students estimate the capacities of containers of a variety of shapes and sizes, paying careful attention to the equal contributions of width, length, and height to the volume. They sort a series of containers from smallest to largest and then check their arrangements by filling the smallest with uncooked rice, pouring that into the second, verifying the fact that it all fits and that more could be added, pouring all of that into the third, and so on.
- Students estimate the angle formed by the hands of the clock. They can also be challenged to find a time when the hands of the clock will make a 90° angle, a 120° angle, or a 180° angle.

4. Explore, construct, and use a variety of estimation strategies.

- Students develop and use the *front end estimation* strategy to obtain an initial estimate of the exact answer. For example, to estimate the total mileage in a driving trip where 354 miles were driven the first day, 412 the second day, and 632 the third, simply add all digits in the hundreds' place: *3 + 4 + 6 = 13 hundred* or 1300.
- Students learn to *adjust* the *front end estimation* strategy to give a more accurate answer. To do so, the second number from the left (in the problem above, the tens) is examined. Here, the estimate would be adjusted up one hundred because the *5 + 1 + 3 tens* are almost another hundred. This would give a better estimate of 1400.
- Students use *rounding* to create estimates, especially in multi-digit addition and subtraction. They do so flexibly, however, rather than according to out-of-context rules. In a grocery store, for example, when a person wants to be sure there is enough money to pay for items that cost $1.89, $2.95, and $4.45, the best strategy may be to *round each price up to the next dollar*. In this case then, the actual sum of the prices is definitely less than $10.00 (*2 + 3 + 5*). On the other hand, to be sure that the total requested by the cashier is approximately correct, the best strategy may be to *round each price to the nearest dollar* and get *2 + 3 + 4* which is $9.
- Students are shown a glass jar filled with about two hundred marbles and are asked to estimate the number in the jar. In small groups, they discuss various approaches to the problem and the strategies they can use. They settle on a strategy to share with the class along with the estimate that resulted.
- Students write about how they might find an estimate for a specific problem in their journals.

5. Recognize when estimation is appropriate, and understand the usefulness of an estimate as distinct from an exact answer.

- Given pairs of real-life situations, students determine which situation in the pair is the one for which estimation is the best approach and which is the one for which an exact answer is probably needed. One such pair, for example, might be: *deciding how much fertilizer is needed for a lawn* and *filling the bags marked "20 pounds" at the fertilizer company*.
- Given a set of cartoons with home-made mathematical captions, third graders decide which of the cartoon characters arrived at exact answers and which got estimates. One of the cartoons might show an adult standing in the checkout line at a supermarket and another

might show the checkout clerk. The captions would read: *Mr. Harris wondered if he had enough money to pay for the groceries he had put in the cart* and *Harry used the cash register to total the bill.* Students make up their own similar cartoon.

- Students share with each other various situations within the past week when they and their families had to do some computation and describe when an exact answer was necessary (and why) and when an estimate was sufficient (and why).

6. Determine the reasonableness of an answer by estimating the result of operations.

- Students are regularly asked if their answer makes sense in the context of the problem they were solving. They respond with full sentences explaining what they were asked to find and why the numerical answer they found fits the context reasonably, that is, why it *could be* the answer.
- Fourth graders might be asked to decide if their estimated answer to the following problem is reasonable. *The band has 103 students in it. They line up in 9 rows. How many students are there in each row?* The students' responses might indicate, for example, that there should be about 10 students in each row, since 103 is close to 100 and 9 is close to 10.
- Students estimate *reasonable* numbers of times that particular physical feats can be performed in one minute. For example: *How many times can you skip rope in a minute? How many times can you hit the = button on the calculator in a minute? How many times can you blink in a minute? How many times can you write your full name in a minute?* and so on. Other students judge whether the estimates are reasonable or unreasonable and then the tasks are performed and actual counts made. (To determine the number of times the = button is hit in a minute, press +1= so that each time the = button is pressed, the display increases by 1.)
- Third-grade students are given a set of thirty cards with three-digit subtraction problems on them. In one minute, they must sort the cards into two piles: those problems whose answers are greater than 300 and those whose answers are less than 300. The correct answers can be on the backs of the cards to allow self-checking after the task is completed.
- For assessment, fourth-grade students might be given a page of one-digit by multi-digit multiplication problems in a multiple choice format with four possible answers for each problem. Within some time period which is much too short for them to perform the actual computations, students are asked to choose the most reasonable estimate from each set of four answers.

7. Apply estimation in working with quantities, measurement, time, computation, and problem solving.

- Students work through the *Product and Process* lesson that is described in the Introduction to this *Framework*. It challenges the students to form two three-digit numbers using 3, 4, 5, 6, 7, 8 which have the largest product; estimation is used to determine the most reasonable possible choices.
- Students learn about different strategies for estimating by reading *The Jellybean Contest* by Kathy Darling or *Counting on Frank* by Rod Clement.
- Students regularly try to predict the numerical facts presented in books like *In an Average*

Lifetime . . . by Tom Heymann. Using knowledge they have and a whole variety of estimation skills, they predict answers to: *What is the number of times the average American eats in a restaurant in a lifetime? (14,411) What is the total length each human fingernail grows in a lifetime? (77.9 inches)* and *What is the average number of major league baseball games an American attends in a lifetime? (16)*

- Students regularly estimate in situations involving classroom routines. For example, they may estimate the total amount of money that will be collected from the students who are buying lunch on Pizza Day or the number of buses that will be needed to take the whole third and fourth grade on the class trip.
- Students investigate environmental issues using estimation. One possible activity is for them to estimate how many gallons of water are used for various activities each week in their home. (See *Healthy Environment — Healthy Me.*)

Activity	Total # in 1 week		Each time		Total # of gallons used
Take shower or bath	________	x	18 gallons	=	
Flush toilet	________	x	7 gallons	=	
Wash dishes	________	x	10 gallons	=	
Wash clothes	________	x	40 gallons	=	
			Total # of gallons used in 1 week	=	

Students discuss possible reasons for differences among their estimates, and they compute the class total for the number of gallons consumed during that week.

References

Clement, Rod. *Counting on Frank.* Milwaukee, WI: Gareth Stevens Children's Books, 1991.

Darling, Kathy. *The Jellybean Contest.* Champaign, IL: Garrard, 1972.

de Paola, Tomie. *The Popcorn Book*. New York: Holiday House, 1978.

Environmental and Occupational Health Sciences Institute. *Healthy Environment — Healthy Me. Exploring Water Pollution Issues: Fourth Grade.* New Jersey: Rutgers University, 1991.

Heymann, Tom. *In an Average Lifetime* ... New York: Random House, 1991.

On-Line Resources

http://dimacs.rutgers.edu/nj_math_coalition/framework.html/

The *Framework* will be available at this site during Spring 1997. In time, we hope to post additional resources relating to this standard, such as grade-specific activities submitted by New Jersey teachers, and to provide a forum to discuss the *Mathematics Standards*.

Standard 10 — Estimation — Grades 5-6

Overview

In grades 5 and 6, students extend estimation to new types of numbers, including fractions and decimals. As indicated in the K-12 Overview, they continue to work on determining the **reasonableness of results,** using **estimation strategies,** and **applying estimation to measurement, quantities,** and **computation.**

In fifth and sixth grade, estimation and number sense are even more important skills than algorithmic paper-and-pencil computation with multi-digit whole numbers. Students should become masters at applying estimation strategies so that answers displayed on a calculator are instinctively compared to a reasonable range in which the correct answer lies.

The new estimation skills that are also important in fifth and sixth grade are skills in estimating the results of fraction and decimal computations. Even though the study of the concepts and arithmetic operations involving fractions and decimals begins before fifth grade, a great deal of time will be spent on them here. A sample unit on fractions for the sixth-grade level can be found in Chapter 17 of this *Framework*. As students develop an understanding of fractions and decimals and perform operations with them, estimation ought always to be present. Estimation of quantities in fraction or decimal terms and of the results of operations on those numbers is just as important for the mathematically literate adult as the same skills with whole numbers.

Children should understand that, sometimes, an estimate will be an accurate enough number to serve as an answer. At other times, an exact computation will need to be done, either mentally, with paper-and-pencil, or with a calculator to arrive at a more precise answer. Which procedure should be used is dependent on the setting and the problem. Even in cases where exact answers are to be calculated, however, students must understand that it is always a good idea to have an estimate in mind before the actual exact computation is done so that the computed answer can be checked against the estimated one.

Standard 10 — Estimation — Grades 5-6

Indicators and Activities

The cumulative progress indicators for grade 8 appear below in boldface type. Each indicator is followed by activities which illustrate how it can be addressed in the classroom in grades 5 and 6.

Building upon knowledge and skill gained in the preceding grades, experiences in grades 5-6 will be such that all students:

5[*]. **Recognize when estimation is appropriate, and understand the usefulness of an estimate as distinct from an exact answer.**

- Given pairs of real-life situations, students determine which is the one in which estimation is the best approach and which is the one needing an exact answer. For example, one such pair, might be: *planning how long it would take to drive from Boston to New York* and *submitting a bill for mileage to your boss.*
- Students collect data for a week on various situations when they and their families had to do some computation and describe when an exact answer was necessary (and why) and when an estimate was sufficient (and why).
- When doing routine problems, the students are always reminded to consider whether their answers make sense. For instance, in the following problem, an estimate makes much more sense than an exact computation. *Molly Gilbert is the owner of a small apple orchard in South Jersey. She has 19 rows of trees with 12 trees in each row. Last year the average production per tree was 761.3 apples. At that rate, what can she expect the total yield to be this year?* For this problem, an exact computation is certain to be wrong and will also be a number that is very hard to remember or use in further planning.
- Students look for examples of estimation language in their reading and/or in the newspapers.
- Students investigate what decisions at the school are based on estimates (e.g., quantity of food for lunch, ordering of textbooks or supplies, making up class schedules, organizing bus routes) by interviewing school employees and making written and/or oral reports.

6. **Determine the reasonableness of an answer by estimating the result of operations.**

- Students estimate the size of a crowd at a rock concert from a picture. They share all of their various strategies with the rest of the class.
- Students demonstrate their understanding by estimating whether or not they can buy a set of items with a given amount of money. For example: *You have only $10. Explain how you can tell if you have enough money to buy: 4 cans of tuna @ 79¢ each, 2 heads of lettuce @ 89¢ each, and 2 lbs. of cheese @ $2.11 per lb.*

---

[*] Activities are included here for Indicators 5 and 6, which are also listed for grade 4, since the Standards specify that students demonstrate continued progress in these indicators.

- Students place decimal points in multi-digit numbers to make absurd statements more *reasonable*. Some typical statements could be *Mr. Brown averages 2383 miles per gallon when he drives to work, My dog weighs 5876 pounds,* or *Big Burger charges $99 for a large order of french fries.*

8. Develop, apply, and explain a variety of different estimation strategies in problem situations involving quantities and measurement.

- Students develop the concept of a billion by estimating its size relative to one hundred thousand, one million, and so on. For instance, they explore questions like *If a calculator is programmed to repeatedly add 1 to the previous display and it takes about forty hours to reach a million, how long would it take to reach a billion?* Students might relate this to the size of the national debt (now $5 trillion).

- Students estimate the area inside a closed curve in square centimeters and then check the estimate with centimeter graph or grid paper.

- After reading Shel Silverstein's poem "How Many, How Much" and Tom Parker's *Rules of Thumb*, students write their own rules of thumb (e.g., *You should never have homework higher than one inch*.).

- Students make estimates about things that happen in one day at school after reading about some of the data in *In One Day,* by Tom Parker. For example, they estimate: *How much pizza is eaten? How much milk is drunk? How many students go home sick?* or *How many students forget something?* They then interview people around the school or conduct a survey to check their estimates.

- Students develop strategies for estimating sums and differences of fractions as they work with them. For example, a fifth grade class is asked to determine which of the following computation problems have answers greater than 1 without actually performing the calculation; many students' strategies will hinge on comparisons of the given fractions to 1/2.

$$\frac{1}{2} + \frac{3}{4} = \qquad\qquad \frac{1}{3} + \frac{5}{6} = \qquad\qquad 1\frac{5}{16} - \frac{1}{2} =$$

- Students make use of strategies like *clustering* and *compatible numbers* in estimating the results of computations. They recognize that a sum of numbers that are approximately the same, such as 37, 39, and 42, can be replaced in an estimate by the product 3 x 40 (*clustering*). They also know that other computations can be performed easily by changing the numbers to numbers that are closely related to each other, such as changing 468 divided by 9 to 450 divided by 9 (*compatible numbers*).

- Students work through the *Mathematics at Work* lesson that is described in the Introduction to this *Framework*. A parent discusses a problem which her company faces regularly: to determine how large an air conditioner is needed for a particular room. To solve this problem, the company has to estimate the size of the room.

9. **Use equivalent representations of numbers such as fractions, decimals, and percents to facilitate estimation.**

   - Students use fractions, decimals, or mixed numbers interchangeably when one form of a number makes estimation easier than another form. For example, rather than estimating the product 3/5 x 4, students consider 0.6 x 4 which yields a much quicker estimate.
   - In their beginning work with percent, students master the common fraction equivalents for familiar percentages and use fractions for estimation in appropriate situations. For example, an estimate of 65% of 63 can be easily obtained by considering 2/3 of 60.

10. **Determine whether a given estimate is an overestimate or an underestimate.**

   - Students decide, as they discuss each new estimation strategy they learn, whether the strategy is likely to give an overestimate, an underestimate, or neither. For instance, using front-end digits will always give an underestimate; rounding everything up (as one might do to make sure she has enough money to pay for items selected in a grocery store) always gives an overestimate; and ordinary rounding may give either an overestimate or an underestimate.
   - Students frequently use *guess, check, and revise* as a problem solving strategy. With this strategy, the answer to a problem is estimated, then calculations are made using the estimate to see whether this estimate meets the conditions of the problem, and the estimate is then revised upwards or downwards as a result. For example, students are asked to find two consecutive pages in a book the product of whose page numbers is 1260. An initial *guess* might be 30 and 31, the *check* might involve concluding that this product is a little over 900, and, as a result, they might *revise* their estimate upwards.

References

Parker, Tom. *In One Day*. Boston: Houghton Mifflin, 1984.

Parker, Tom. *Rules of Thumb*. Boston: Houghton Mifflin, 1983.

Silverstein, Shel. “How Many, How Much,” in *A Light in the Attic*. New York: Harper and Row, 1981.

On-Line Resources

http://dimacs.rutgers.edu/nj_math_coalition/framework.html/

The *Framework* will be available at this site during Spring 1997. In time, we hope to post additional resources relating to this standard, such as grade-specific activities submitted by New Jersey teachers, and to provide a forum to discuss the *Mathematics Standards*.

Standard 10 — Estimation — Grades 7-8

Overview

Estimation, as described in the K-12 Overview includes three primary themes: determining the **reasonableness of answers**, using a variety of **estimation strategies** in a variety of situations, and **estimating the results of computations**.

In seventh and eighth grade, estimation and number sense are much more important skills than algorithmic paper-and-pencil computation with whole numbers. Students should become masters at applying estimation strategies so that an answer displayed on a calculator is instinctively compared to a reasonable range in which the correct answer lies. It is critical that students understand the displays that occur on the screen and the effects of calculator rounding either because of the calculator's own operational system or because of user-defined constraints. Issues of the number of significant figures and what kinds of answers make sense in a given problem setting create new reasons to focus on **reasonableness of answers**.

The new estimation skills begun in fifth and sixth grade are still being developed in the seventh and eighth grades. These include skills in estimating the results of fraction and decimal computations. As students deepen their understanding of these numbers and perform operations with them, estimation ought always to be present. Estimation of quantities in fraction or decimal terms as a result of operations on those numbers is just as important for the mathematically literate adult as the same skills with whole numbers.

In addition, the seventh and eighth grades present students with opportunities to develop strategies for estimation with ratios, proportions, and percents. Estimation and number sense must play an important role in the lessons dealing with these concepts so that students feel comfortable with the relative effects of operations on them. Another new opportunity here is estimation of roots. It should be well within every eighth grader's ability, for example, to estimate the square root of 40.

Students should understand that sometimes, an estimate will be accurate enough to serve as an answer. At other times, an exact computation will need to be done, either mentally, with paper-and-pencil, or with a calculator. Even in cases where exact answers are to be calculated, however, students must understand that it is almost always a good idea to have an estimate in mind so that the computed answer can be checked against it.

Standard 10 — Estimation — Grades 7-8

Indicators and Activities

The cumulative progress indicators for grade 8 appear below in boldface type. Each indicator is followed by activities which illustrate how it can be addressed in the classroom in grades 7 and 8.

Building upon knowledge and skills gained in the preceding grades, experiences in grades 7-8 will be such that all students:

5*. Recognize when estimation is appropriate, and understand the usefulness of an estimate as distinct from an exact answer.

- Students regularly tackle problems for which estimation is the only possible approach. For example: *How many hairs are on your head?* or *How many grains of rice are in this ten-pound bag?* Solution strategies are always discussed with the whole class.
- Students create a plan to "win a contract" by bidding on projects. For example: *Your class has been given one day to sell peanuts at Shea Stadium. Prepare a presentation that includes the amount of peanuts to order, the costs of selling the peanuts, the profits that will be made, and the other logistics of selling the peanuts. Organize a schedule with estimated times for completion for the entire project.*
- Students use estimation skills to run a business using *Hot Dog Stand* or *Survival Math* software.
- Students apply estimation skills to algebraic situation as they try to guess the equations to hit the most globs in *Green Globs* software.

6. Determine the reasonableness of an answer by estimating the result of operations.

- Students estimate whether or not they can buy a set of items with a given amount of money. For example: *I have only $50. Can I buy a reel, a rod, and a tackle box during the sale advertised below?*

  ALL ITEMS 1/3 OFF AT JAKE'S FISHING WORLD!

ITEM	REGULAR PRICE
Daiwa Reels	$29.95 each
Ugly Stick Rods	$20.00 each
Tackle Boxes	$17.99 each

  To assess students' performance, the teacher asks them to write about how they can answer this question without doing any exact computations.

  If 6% sales tax is charged, can you tell whether $50 is enough by estimating? Explain. Calculate the exact price including tax.

---

* Activities are included here for Indicators 5 and 6, which are also listed for grade 4, since the Standards specify that students demonstrate continued progress in these indicators.

- Students evaluate various statements made by public figures to decide whether they are reasonable. For example: *The Phillies' center fielder announced that he expected to get 225 hits this season. Do you think he will?* In order to determine what confidence to have in that prediction, a variety of factors need to be estimated: number of at-bats, lifetime batting average, likelihood of injury, whether a baseball strike will occur, and so on.

- Students discuss events in their lives that might have the following likelihoods of occurring:

  100%, 0.5 %, 3/4, 95%

- Students simulate estimating the number of fish in a lake by estimating the number of fish crackers in a box using the following method. Some of the fish are removed from the box and "tagged" by marking them with food coloring. They are "released" to "swim" as they are mixed in with the other crackers in the box. Another sample is drawn and the number of tagged fish and the total number of fish are recorded. This data is used to set up a proportion (# tagged initially: total # fish = # tagged caught: total # caught) to predict the total number of fish in the box. The estimate is further improved by taking additional samples, making predictions based on each sample of the total number of fish in the box, and finally averaging all the predictions.

8. Develop, apply, and explain a variety of different estimation strategies in problem situations involving quantities and measurement.

- Students regularly have opportunities to estimate answers to straight-forward computation problems and to discuss the strategies they use in making the estimations. Even relatively routine problems generate interesting discussions and a greater shared number sense within the class:

  23% of 123, *5* x *38,* *28* x 425, 486 x 2004, 423÷71

- Given a ream of paper, students work in small groups to estimate the thickness of one sheet of paper. Answers and strategies are compared across groups and explanations for differences in the estimates are sought. To assess student understanding, the teacher asks each student to write about how his or her group solved the problem.

- Students develop strategies for estimating the results of operations on fractions as they work with them. For example, a seventh grade class is asked to determine which of these computation problems have answers greater than 1 without actually performing the calculation:

  $\frac{1}{2} \times \frac{3}{4} =$ $\qquad$ $\frac{5}{6} \div \frac{1}{3} =$ $\qquad$ $\frac{1}{2} \div 1\frac{1}{2} =$

- Students make estimates of the number of answering machines, cellular phones, and fax machines in the United States and check their results against data from *America by the Numbers* by Les Krantz.

- Students estimate the total number of people attending National Football League games by determining how many teams there are, how many games each plays, and what the average attendance at a game might be. They then use these estimates to determine the overall answer (17,024,000 according to *America by the Numbers,* p. 194).

- Students determine the amount of paper thrown away at their school each week (or month or year) by collecting the paper thrown away in their math class for one day and multiplying this by the number of classes in the school and then by five (or 30 or 50).

9. Use equivalent representations of numbers such as fractions, decimals, and percents to facilitate estimation.

- Students use fractions, decimals, or mixed numbers interchangeably when one form of the number makes estimation easier than another. For example, rather than estimating $33\frac{1}{3}$% of \$120, students consider 1/3 x 120 which yields a much quicker estimate.
- Similarly, in their work with percents, students master the common fraction equivalents for familiar percentages and use fractions for estimation in appropriate situations. For example, an estimate of 117% of 50 can most easily be obtained by considering 6/5 of 50.
- Students collect and bring to class sales circulars from local papers which express the discounts on sale items in a variety of ways including percent off, fraction off, and dollar amount off. For items chosen from the circular, the students discuss which form is the easiest form of expression of the discount, which is most understandable to the consumer, and which makes the sale seem the biggest bargain.

10. Determine whether a given estimate is an overestimate or an underestimate.

- Using calculators, but without using the square root key, students try to find good approximations for a few square roots, for example, the square root of 40. Through a series of approximations, they make a guess, perform the multiplication on the calculator, determine whether the approximation was too large or too small, adjust it, and begin again. This series of approximations, in itself a very useful strategy, continues until an approximation is reached that is satisfactory.
- Students compare three different rounding strategies:

  Round everything up: for example, 2345, rounded to the nearest ten, would be 2350. To the nearest hundred, it would be 2400.

  Round up if 5 or more, down if less than 5: for example, 2345, rounded to the nearest ten, would be 2350. To the nearest hundred, it would be 2300.

  Round up if more than 5, down if less than 5; and for the special case when the digit equals five make the preceding digit even. That is, if the number before the five is odd, round up; if it is even, round down. For example, 2345, rounded to the nearest ten, would be 2340. To the nearest hundred, it would be 2300.

  They discuss whether each strategy would be more likely to yield an overestimate or an underestimate when adding up a total and then apply all these strategies to several situations.

References

Krantz, Les. *America by the Numbers.* New York: Houghton Mifflin, 1993. (Note: Some of the entries in this book are unsuitable for seventh- and eighth-graders.)

Software

Green Globs and *Graphing Equations*. Sunburst Communications.

Hot Dog Stand. Sunburst Communications.

Survival Math. Sunburst Communications.

On-Line Resources

http://dimacs.rutgers.edu/nj_math_coalition/framework.html/

The *Framework* will be available at this site during Spring 1997. In time, we hope to post additional resources relating to this standard, such as grade-specific activities submitted by New Jersey teachers, and to provide a forum to discuss the *Mathematics Standards*.

Standard 10 — Estimation — Grades 9-12

Overview

Estimation is a combination of content and process. Students' abilities to use estimation appropriately in their daily lives develop as they have regular opportunities to explore and construct estimation strategies and as they acquire an appreciation of its usefulness through using estimation in the solution of problems. At the high school level, estimation includes focusing on the **reasonableness of answers** and using various **estimation strategies for measurement, quantity, and computations**.

In the high school grades, estimation and number sense are much more important skills than algorithmic paper-and-pencil computation. Students need to be able to judge whether answers displayed on a calculator are within an acceptable range. They need to understand the displays that occur on the screen and the effects of calculator rounding either because of the calculator's own operational system or because of user-defined constraints. Issues of the number of significant digits and what kinds of answers make sense in a given problem setting create new reasons to a focus on **reasonableness of answers.**

Measurement settings are rich with opportunities to develop an understanding that estimates are often used to determine approximate values which are then used in computations and that results so obtained are not exact but fall within **a range of tolerance**. Appropriate issues for discussion at this level include acceptable limits of tolerance, and assessments of the degree of error of any particular measurement or computation.

Another topic appropriate at these grade levels is the estimation of probabilities and of statistical phenomena like measures of central tendency or variance. When statisticians talk about "eyeballing" the data, they are explicitly referring to the process where these kinds of measures are estimated from a set of data. The skill to be able to do that is partly the result of knowledge of the measures themselves and partly the result of experience in computing them.

Standard 10 — Estimation — Grades 9-12

Indicators and Activities

The cumulative progress indicators for grade 12 appear below in boldface type. Each indicator is followed by activities which illustrate how it can be addressed in the classroom in grades 9, 10, 11, and 12.

Building upon knowledge and skills gained in the preceding grades, experience in grades 9-12 will be such that all students:

6[*]. Determine the reasonableness of an answer by estimating the result of operations.

- Students are routinely asked if the answers they've computed make sense. *Latisha's calculator displayed 17.5 after she entered 3 times the square root of 5. Is this a reasonable answer?*
- Students are sometimes presented with hypothetical scenarios that challenge both their estimation and technology skills: *During a test, Paul entered $y = .516x - 2$ and $y = .536x + 5$ in his graphics calculator. After analyzing the two lines displayed on the "standard" screen window setting [-10,10, -10,10], he decided to indicate that the lines were parallel and that there was no point of intersection. Was Paul's answer reasonable?*
- On a test, students are asked the following question: *Jim used the zoom feature of his graphics calculator and found the solution to the system $y = 2x + 3$ and $y = -2x - 1$ to be (-.9997062,1.0005875). When he got his test back his teacher had taken points off. What was wrong with Jim's answer?*

11. Estimate probabilities and predict outcomes from real-world data.

- Students use tables of data from an almanac to make estimates of the means and medians of a variety of measures such as the average state population or the average percentage of voters in presidential elections. Any table where a list of figures (but no mean) is given can be used for this kind of activity. After estimates are given, actual means and medians can be computed and compared to the estimates. Reasons for large differences between the means and medians ought to also be explored.
- Students collect data about themselves and their families for a statistics unit on standard deviation. After everyone has entered data in a large class chart regarding number of siblings, distance lived away from school, oldest sibling, and many other pieces of numerical data, the students work in groups to first estimate and then compute means and medians as a first step toward a discussion of variation.
- Students each track the performance of a particular local athlete over a period of a few weeks and use whatever knowledge they have about past performance to predict his or her performance for the following week. They provide as detailed and statistical a prediction as

[*]Activities are included here for Indicator 6, which is also listed for grade 8, since the Standards specify that students demonstrate continued progress in this indicator.

possible. At the end of the week, predictions are compared to the actual performance. Written reports are evaluated by the teacher.

12. Recognize the limitations of estimation, assess the amount of error resulting from estimation, and determine whether the error is within acceptable tolerance limits.

- Students in high school learn methods for estimating the magnitude of error in their estimations at the same time as they learn the actual computational procedures. Discussions regarding the acceptability of a given magnitude of error are a regular part of classroom activities when estimation is being used.

- Students work in small groups to carefully measure the linear dimensions of a rectangular box and determine its volume using measures to the nearest 1/8 inch or the smallest unit on their rulers. After their best measurements and computation, the groups share their estimates of the volume and discuss differences. Each group then constructs a range in which they are sure the exact answer lies by first using a measure for each dimension which is clearly short of the actual measure and multiplying them, and then by finding a measure for each dimension which is clearly longer than the actual measure and multiplying those. The exact answer then lies between those two products. Each group prepares a written report outlining their procedures and results.

- Students are presented with these two solutions to the following problem and discuss the error associated with each approach: *How many kernels of popcorn are in a cubic foot of popcorn?*
  1. *There are between 3 and 4 kernels of popcorn in 1 cubic inch. There are 1728 cubic inches in a cubic foot. Therefore there are 6048 kernels of popcorn in a cubic foot. [3.5 (the average number of kernels in a cubic inch) x 1728 = 6048]*
  2. *The diameter of a kernel of popcorn is approximately 9/16 of an inch. The volume of this "sphere" is 0.09314 cubic inches. Therefore (1728/0.09314) = 18552.71634 or 18,000 pieces of popcorn.*

- Students write a computer program to round any number to the nearest hundredth.

- Students analyze the error involved in rounding to any value. For example, a number rounded to the nearest ten, say 840, falls into this range: $835 \le X < 845$. The error involved could be as large as 5. Similarly, a number rounded to the nearest hundredth, say .84, falls into the range: $0.835 \le X < 0.845$. The error could be as large as .005.

- Students discuss what is meant by the following specifications for the diameter of an 0-ring: 2.34 ± 0.005 centimeters.

- Students act as *quality assurance officers* for mythical companies and devise procedures to keep errors within acceptable ranges. One possible scenario:

  In order to control the quality of their product, Paco's Perfect Potato Chip Company guarantees that there will never be more than 1 burned potato chip for every thousand that are produced. The company packages the potato chips in bags that hold about 333 chips. Each hour 9 bags are randomly taken from the production line and checked for burnt chips. If more than 15 burnt chips are found within a four hour shift, steps are taken to reduce the number of burnt chips in each batch of chips produced. Will this plan ensure the company's guarantee?

- Students regularly review statistical claims reported in the media to see whether they accurately reflect the data that is provided. For example, did the editor make appropriate use of the data given below? (Based on an example in *Exploring Surveys and Information from Samples* by James Landwehr.)

  The March, 1985 Gallup Survey asked 1,571 American adults "Do you approve or disapprove of the way Ronald Reagan is handling his job as president?" 56% said that they approved. For results based on samples of this size, one can say with 95% confidence that the error attributable to sampling and other random effects could be as much as 3 percentage points in either direction. A newspaper editor read the Gallup survey report and created the following headline: BARELY ONE-HALF OF AMERICA APPROVES OF THE JOB REAGAN IS DOING AS PRESIDENT.

References

Landwehr, James. *Exploring Surveys and Information from Samples.* Palo Alto, CA: Dale Seymour, 1987.

On-Line Resources

http://dimacs.rutgers.edu/nj_math_coalition/framework.html/

The *Framework* will be available at this site during Spring 1997. In time, we hope to post additional resources relating to this standard, such as grade-specific activities submitted by New Jersey teachers, and to provide a forum to discuss the *Mathematics Standards*.